Recursion Time Complexity

Factorial

5! (BP) 4! * 5 (SP) (Self work)

```
int fact (int n)            T(n)
{   if(n==0) return 1;
    int sp= fact(n-1);      T(n-1)
    int ans= sp*n;
    return ans;
}
```

Recurrence Relation

$$T(n) = T(\cancel{n-1}) + 1$$

$$T(\cancel{n-1}) = T(\cancel{n-2}) + 1$$

$$T(\cancel{n-2}) = T(\cancel{n-3}) + 1$$

$$\vdots$$

$$T(\cancel{2}) = T(\cancel{1}) + 1$$

$$T(\cancel{1}) = 1$$

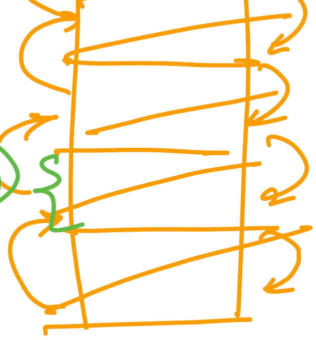
$$T(n) + T(\cancel{n-1}) + T(\cancel{n-2}) \ldots T(\cancel{1}) = T(\cancel{n-1}) + 1 + T(\cancel{n-2}) + 1 + T(n-3) + 1 \ldots T(\cancel{1}) + 1 + 1$$

$$\left.\begin{matrix} T(n) = n-1 \\ n-2 \end{matrix}\right\} O(n)$$

Shortcut: Single Rec Call

$O(1)$

$sp^{*}n$



Time ?

$$\frac{\frac{\text{no of fn}^n \text{ frame} * \text{time in each frame}}{n \quad * \quad 1}}{n}$$

Fibonacci

```
int fib(n)                    T(n)
{
    if(n==0||n==1) return n;
    int fnm1 = fib(n-1);      T(n-1)
    int fnm2 = fib(n-2);      T(n-2)
    int fn = fnm1 + fnm2;   } constant
    return fn;
}
```

$T(n) = T(n-1) + T(n-2) + 1 \le T(n-1) + T(n-1) + 1$

$T(n) \le 2\cancel{T(n-1)} + 1$

$2\cancel{T(n-1)} \le 2^2 \cancel{T(n-2)} + 1 \cdot 2 \quad \times 2^1$

$2^2 \cancel{T(n-2)} \le 2^3 \cancel{T(n-3)} + 1 \cdot 2^2 \quad \times 2^2$

$\vdots \qquad \vdots$

$2^{n-2} T(n-\cancel{(n-2)}) \le 2^{n-1} \cancel{T(1)} + 1 \cdot 2^{n-2} \quad \times 2^{n-2}$

$2^{n-1} T(n-\cancel{(n-1)}) = 1 \cdot 2^{n-1} \quad \times 2^{n-1}$

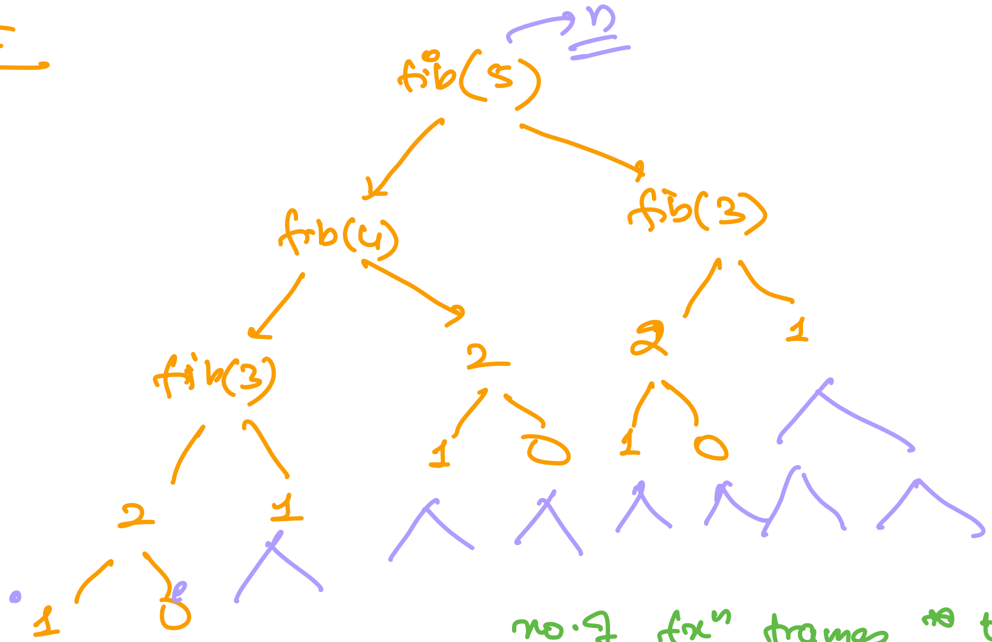
$a\left(\frac{r^n - 1}{r-1}\right)$

$T(n) = 1 + 2 + 2^2 + \cdots \; 2^{n-2} + 2^{n-1}$

$T(n) = \left(\frac{2^n - 1}{2-1}\right)$

$T(n) = 2^n - 1 = O(2^n)$

Shortcut



2^0

2^1

2^2

2^3

2^4

$2^0+2^1+2^2+2^3+2^4 = 2^5-1$

no. of fx^n frames * time taken in 1 fx^n work

$= (2^n - 1) * 1$

$= O(2^n)$

tree ht

no of calls

≥ 2 calls:

calls^{ht} * fx^n frame self work

Q: x^n Power

$x^n = x^{n-1} * x$

BP SP SW

$T(n) = T(n-1) + 1 \Big\} O(n)$

SC:

$(n+1) * 1 = O(n)$?

2^{10}
↓
2^9
↓
2^8

better?

$2^{10} = 2^5 \cdot 2^5$

$2^{11} = 2^5 \cdot 2^5 \cdot 2$

$$2^7 \downarrow 2^6 \downarrow 2^5 \downarrow 2^4 \downarrow 2^2 \downarrow 2^2 \downarrow 2 \downarrow 2^0$$

$$2^5 = 2^2 \cdot 2^2 \cdot 2$$

$$2^2 = 2 \cdot 2$$

$$2^1 = 2^0 \cdot 2^0 \cdot 2$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + 1$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$2T\left(\frac{n}{2}\right) = 2^2 T\left(\frac{n}{4}\right) + 1 \cdot 2$$

$$2^2 T\left(\frac{n}{4}\right) = 2^3 T\left(\frac{n}{8}\right) + 1 \cdot 2^2$$

$$\vdots$$

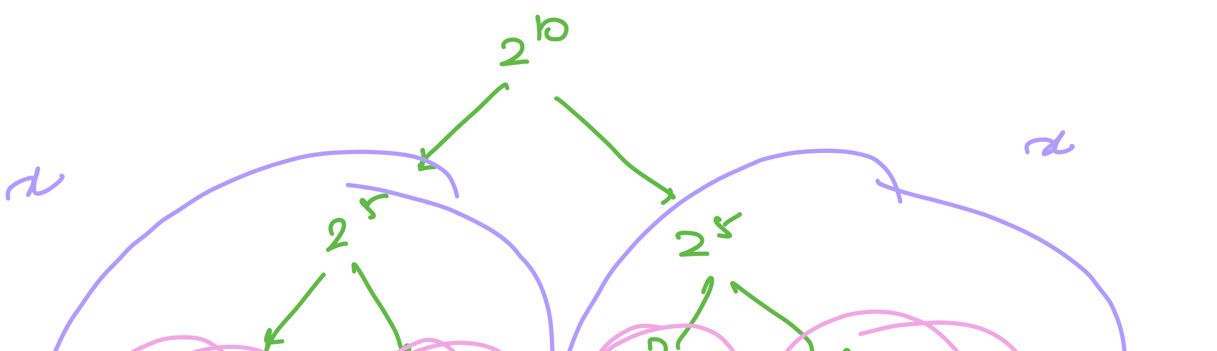
$$n = 2^{\log_2 n}$$

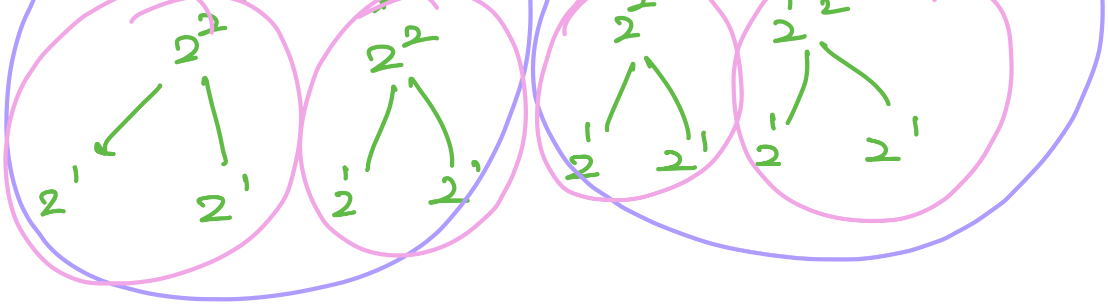
$$2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) = 1 \cdot 2^{\log_2 n}$$

$$T(n) = 1 + 2 + 2^2 + \cdots\cdots 2^{\log_2 n}$$

$$T(n) = \underbrace{2^{\log_2 n + 1} - 1}_{f(n)} \le 2^{\log_2 n + 1} \le \underbrace{3}_{c} \cdot \underbrace{2^{\log_2 n}}_{g(n)}$$

$$= O\left(2^{\log_2 n}\right) = O(n)$$





$$\left.\begin{aligned}
T(n) &= T\cancel{\left(\tfrac{n}{2}\right)} + 1 \\
T\cancel{\left(\tfrac{n}{2}\right)} &= T\cancel{\left(\tfrac{n}{4}\right)} + 1 \\
T\cancel{\left(\tfrac{n}{4}\right)} &= T\cancel{\left(\tfrac{n}{8}\right)} + 1 \\
T\cancel{\left(\tfrac{n}{8}\right)} &= T\cancel{\left(\tfrac{n}{16}\right)} + 1 \\
&\vdots \\
T\cancel{(1)} &= 1
\end{aligned}\right\} \log n$$

$$\underbrace{1+1+1 \text{ ——— } 1}_{\log n}$$

$$O(\log n)$$

Program $\rightarrow$ TC?

online ~~Platform~~
Judge:

Constraints: $\left.\begin{aligned} 0 &\le n \le 10^5 \\ 0 &\le x \le 20 \end{aligned}\right\}$

Bubble Sort : n^2

Array: $n = 10^5$

Instruction Ex : $(10^5)^2 = \underline{\underline{10^{10}}}$

1 GHz

10^9 instn in 1 sec

1 ins $\rightarrow \frac{1}{10^9}$ sec

10^{10} ins $\rightarrow \frac{10^{10}}{10^9}$ sec $=$ 10 sec

Input size	Complexity
$n \le 10$	$n!$ 2^n
$n \le 100$	n^4
$n \le 400$	n^3
$n \le 2000$	$n^2 \log n$
$n \le 10^4$	n^2
$n \le 10^6$	$n \log n$
$n \le 10^8$	n
$n \le 10^{18}$	$\log n$

$$1 < \log\log n < \sqrt{\log n} < \log n < \sqrt{n} < n < n\log n < n^2 < n^2\log n < n^2 <$$
$$n^2 \log n < c^n < n! < n^n$$

Inc order of TC